A Long-Run Productivity Risks Driving q-Factor Model

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This is a very primary draft, please feel free to draw comments.

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Abstract

I incorporate the productivity risks into an investment-based q-factor asset pricing model. The productivity risks factors largely summarize the cross-sectional portfolio return, in which the time-varying volatility plays an important role. A parsimonious q-factor model driven by productivity risks explains about 90% variation of return of 25 Size/BM portfolios and around 75% variation of return of 160 portfolios, which is comparable to the Fama-French multifactor models, the Carhart (1997) four-factor model, and the Hou, Mo, Xue & Zhang (2020) q^5 model. As such, productivity growth risks can be one of the potential forces driving investment-based factor models.

Key Words: The q-Factor Model, Investment-Based Asset Pricing, Long-Run Productivity Risks, Markov Regime Switching.

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1 Introduction

A growing literature focuses on the economic background behind the multifactor asset pricing models under the investment approach (see Hou et al. 2015, 2019, Hou, Xue & Zhang 2020, Hou, Mo, Xue & Zhang 2020, among others). However, the economic driving forces behind such investment-based asset pricing models have received little attention. The major contribution of this paper is to show that productivity risks, in particular the productivity uncertainty risks, are the potential driving forces behind the investment-based q-factor model. To this end, I show that like the Hou et al. (2015)'s q-Factor model and Hou, Mo, Xue & Zhang (2020)'s q^5 model, the major productivity risk factors can be derived from the first-order condition of investment return, which naturally integrates productivity risk factors to the investment-based q-factor model. Besides, this paper shows that the exposure to productivity risks, in particular the volatility risks, leads to considerable risk premiums. As such, aggregate productivity risks conduce to understand the empirical success of q-Factor series models.

In particular, productivity growth uncertainty plays an important role in driving the investment-based q-factor model. It has long been recognised that the macroeconomic volatility risks impact on asset prices. However, the previous literature of the production and investment-based asset pricing in long-run risks address quantitative models (see Caldara et al. 2012, Kaltenbrunner & Lochstoer 2010, Croce 2014, among others), only a little work put on empirical asset pricing tests (Chen et al. 2020). As such, I empirically specify the time-varying productivity uncertainty process and find that it is the crucial factor in determining the equity return variations. As such, this paper brings the literature of long-run risks and investment-based factor pricing model together.

My empirical examination of macroeconomic uncertainty starts with following the work of Boguth & Kuehn (2013). I assume the first and second-order conditional moments of its productivity shock follow Markov regime-switching processes, whose persistences capture the long-run risks and time-varying macro volatility. I then follow Hamilton (1990) and Liu & Miao (2015)'s Expectation Maximisation algorithm to identify the hidden Markov chains for the first and second-order conditional moments of productivity growth shock. Using the samples of productivity shock from 1963Q1 to 2020Q2, I find that the empirical estimation strongly supports the presence of shifts in mean and volatility regimes, with mean regimes being more persistent.

In my main asset pricing tests, I propose two benchmark models that can incorporate productivity risks into an equilibrium framework. The first benchmark model only combines all productivity risk factor with an additional investment-capital ratio factor, which can entirely be derived from the first-order condition of investment return. The second benchmark model includes all factors from the first benchmark model, with the additional market risk premiums factor and size factor from the q-factor model. I investigate the fitness and standard errors of my benchmark asset pricing models using annual returns of 160 portfolios (i.e., 25 portfolios sorted on size/value, 25 portfolios sorted on size/profitability, 25 portfolios sorted on size/investment, 25 portfolios sorted on value/profitability, 25 portfolios sorted on value/investment, 25 portfolios sorted on profitability/investment, 10 portfolios sorted on industries). I consider various equity portfolios and an investment-based theoretical framework to avoid the critique of Lewellen et al. (2010)[2].

The tests results are striking. The first benchmark model merely consisting of the productivity risks and investment capital ratio captures around 90% stock variation of the equal-weighted 25 SIZE/BM portfolios, which outperforms most of factor pricing models. It also explains the three-fourths equity variation of equal-weighted 160 portfolios, which is comparable to the majority factor pricing models. The second benchmark model even has a better performance than the first one. Despite the test performances of both benchmark

[2]Lewellen et al. (2010) suggest that testing multifactor asset pricing models with 25 SIZE/BM portfolios and evaluating model performances with cross-sectional R^2 is highly misleading. When adding industry portfolios, the cross-sectional R^2 sharply reduce.

models drop in the sample of value-weighted portfolios, the second benchmark model is still the leader in a group of asset pricing models. By subtracting the risk factors of productivity growth and its conditional mean, I find that productivity uncertainty is a negatively priced source that plays a major role in explaining the cross-sectional equity return variation. To end, I show that the empirical success of the productivity shock is based on it captures the feature of bad economic states, as the high productivity volatility occurs in bad times.

My work is highly close to the work of Chen et al. (2020) but has several major differences. First, their work links to the first-order condition of investment return by the expected growth factor in Hou, Mo, Xue & Zhang (2020)'s q^5 model. In their work, the productivity risks belong to the capital gain rather than the dividend yield. By contrast, my work mathematically illustrates that, in an equilibrium framework, the TFP risks belong to the dividend yield component (i.e., marginal production function to capital) rather than the item of capital gain, given the first-order condition of the firm maximisation problem. The stochastic process driving the law of motion of capital accumulation (i.e., the investment rate growth) is viewed as the investment-specific technology shock in the standard macroeconomic literature (see Fisher 2006, Justiniano et al. 2010, Winberry 2018, among others) rather than the TFP process. Thus, my work incorporates the productivity risks into the investment-based model with a more rigorously macroeconomic approach.

Furthermore, Chen et al. (2020) estimate the productivity risks from Compustat financial data, but this paper chooses the TFP samples from the San Francisco Fed. Besides, Chen et al. (2020) selected the macro uncertainty risk factors by statistical criteria. As such, they can hardly use the macro uncertainty factor to explain the profitability factor, because the correlation between macro uncertainty and ROE factor is too low in their work [3]. In contrast,

[3] Chen et al. (2020) report that their macro uncertainty factor is highly correlated to the expected growth factor with a correlation coefficient 0.22, but it almost has no relation to the profitability factor, as the correlation coefficient is -0.02. My work reports that the correlation coefficient between macro uncertainty and profitability is 0.21.

I follow Boguth & Kuehn (2013) to specify the productivity growth follows a four-state Markov chain and construct the related risk factors, whose correlation to profitability factor is relatively high, which is not the case in Chen et al. (2020)'s work.

Moreover, Chen et al. (2020) examine asset pricing implications behind both macro uncertainty and micro uncertainty in which the micro uncertainty plays a marginal role. As such, my work only focuses on the macro uncertainty and show that it largely captures the variation of cross-sectional stock return. An additional difference is I consider both first and second-order perceived moments of long-run productivity as risk factors while Chen et al. (2020) only consider the second-order moment. Despite such differences, both studies suggest that macro uncertainty leads to sizable risk premiums and provides insight to understand the q-factor model. However, my work is more close to the economic equilibrium framework than Chen et al. (2020)'s work.

In sum, my work provides the insights behind the q-Factor model and q^5 model in two points. First, the productivity risks summarise the features of economic recession and sufficiently explains the stock return variation in the cross-section. As such, productivity risks link the macroeconomic fluctuations to the equity return. Second, both productivity risks and q-factor model are derived from the first-order condition of investment return such that productivity risks are naturally involved in the q-factor model. Therefore, productivity risks could be possible sources to drive the investment-based q-factor asset pricing model.

Literature Review This study contributes to three strands of the literature. First, this study contributes to the asset pricing implications of aggregate productivity volatility risks. The stochastic volatility has been widely studied in the macro-asset pricing literature, in particular, the consumption-based models (see Bansal & Yaron 2004, Bansal et al. 2005, Boguth & Kuehn 2013, among others). In the production side, Caldara et al. (2012), Kaltenbrunner & Lochstoer (2010), and Croce (2014) build quantitative models with stochastic volatility in productivity to investigate the asset pricing puzzles. Empirically, Chen et al.

(2020) investigates the asset prices effects of uncertainty shock by explaining the expected growth factor. In contrast, my work addresses the effects of productivity risks by seeking the driving force behind the first-order condition of production.

Further, this research belongs to the empirical literature of investment-based asset pricing model. Cochrane (1991) proves that the investment return equals the equity return. As such, the aggregate investment responds to equity risk premiums, which links asset prices to business cycle fluctuations. One step further, Cochrane (1996) shows that the investment capital ratio is a risk factor that can substantially explain the cross-sectional stock return. Belo (2010) links a flexible production technology to the cross-sectional portfolio return. My work contributes to this branch of literature by specifying a detailed productivity process from the economic data.

Moreover, this paper offers a potential economic background behind the multifactor asset pricing models. On the one hand, Fama & French (1993, 1996, 2015, 2018) consider the Inter-temporal CAPM model (Merton 1973) as the theoretical background. On the other hand, Hou et al. (2015) and Hou, Mo, Xue & Zhang (2020) explain multifactor models by the investment-based asset pricing framework. However, Hou, Mo, Xue & Zhang (2020) points out that further studies should shed light on the economic driving forces of the risk factors(e.g., profitability, expected growth, investment). My work adds in this strand of literature by introducing productivity risks. As such, my work could be considered as a supplement of Hou et al. (2015)'s q-factor model, or an alternative of Hou, Mo, Xue & Zhang (2020) for I provide another approach to extend a static model to its dynamic version.

Road Map The rest of this paper is organized as follows. Section 2 presents a parsimonious investment-based asset pricing model. Section 3 introduces the identification methodologies, which includes Hamilton (1990)'s Expectation Maximisation algorithm and Fama & Macbeth (1973) two-pass regression. Section 4 illustrates all empirical results. Section 5 draws conclusions.

2 An Investment-Based Asset Pricing Framework

2.1 Model Setup

Environment Assume the representative firm lives in an infinite horizon environment with discrete-time as Cochrane (1996). The firm adopts the physical capital and technology to produce a single investment good and maximise its profits.

Technology The production sector follows an investment-based model with a parsimonious assumption that constant labour $N_t = 1$:

$$Y_t = A_t K_t^{\alpha} \tag{1}$$

where Y_t and K_t denote the aggregate output and capital stock. The aggregate productivity A_t follows a four-state Markov Chains process.

Markov Regime Switching in Productivity I assume both the first and second-order moment of the aggregate productivity growth follow the Markov Chain. More specifically, the logarithm productivity growth $\log \frac{A_{t+1}}{A_t}$ follows:

$$\log \frac{A_{t+1}}{A_t} = \Delta a_t = \nu_t + \sigma_t \epsilon_t, \epsilon_t \sim N(0, 1). \tag{2}$$

in which ν_t and σ_t present the conditional mean and standard deviation. Following Boguth & Kuehn (2013), I assume two states for the conditional mean $\nu_t \in \{\nu_h, \nu_l\}$, and other two independent states for the conditional volatility $\sigma_t \in \{\sigma_h, \sigma_l\}$ where h and l denote the high and low level, respectively. Specifically, the economy has probability p^{μ}_{ll} (p^{μ}_{hh}) to stay a low (high) growth rate if it enters in a low (high) growth rate at the beginning. Second, the economy has probability p^{σ}_{ll} (p^{σ}_{hh}) to face a low (high) macro uncertainty state if it starts with a low (high) macro uncertainty state. As such, the transition matrices P^{μ} and P^{σ} is denoted by:

$$P^{\mu} = \begin{bmatrix} p_{ll}^{\mu} & 1 - p_{ll}^{\mu} \\ 1 - p_{hh}^{\mu} & p_{hh}^{\mu} \end{bmatrix}, P^{\sigma} = \begin{bmatrix} p_{ll}^{\sigma} & 1 - p_{ll}^{\sigma} \\ 1 - p_{hh}^{\sigma} & p_{hh}^{\sigma} \end{bmatrix}. \tag{3}$$

Given the independent assumption above, the joint transition matrix is the product of transition matrices of conditional expectation and standard deviation, which consists of 16 elements. Note that the Hidden Markov Chain processes are unobservable, the agent will infer the future state from observable data Y_t and K_t. Like Boguth & Kuehn (2013), I consider $\theta_{t+1|t}$ as current period's prior belief vector for the future period's states:

$$\theta_{t+1|t} = P' \frac{\theta_{t|t-1} \odot \eta_t}{1' \left(\theta_{t|t-1} \odot \eta_t\right)} \tag{4}$$

in which $P' = P^{\mu} \otimes P^{\sigma}$ and η_t represent the joint transition matrix and the vector of conditional Guassian likelihood functions, respectively. In this case, $\odot$ and $\otimes$ denote the element-by-element multiplication and Kronecker tensor product.

Capital Adjustment and Accumulation The law of motion of capital accumulation process bases on the standard investment-based asset pricing model with one additional ingredient—the convex capital adjustment costs. I present the capital accumulation process as:

$$K_{t+1} = \left\{ (1 - \delta) K_t + \frac{\phi}{2} \left(\frac{I_t}{K_t} \right)^2 K_t \right\} \tag{5}$$

where ϕ is a parameter to control the convex capital adjustment costs.

2.2 Asset Pricing Implications

Let me state the ex-dividend equity price as P_t and the dividend as D_t in current period. Taking the stochastic discount factor $M_{t,t+1}$ as given, on the production block, the firm chooses the optimal investment stream to maximise the market equity, which satisfies the first principle of investment $E_t\left[M_{t,t+1}R^I_{t+1}\right]=1$. As such, the firm problem is denoted by:

$$V_t=\max_{I_t}E_0\left\{\sum_{t=0}^{\infty}M_{0,t}\left[Y_t-\frac{\phi}{2}\left(\frac{I_t}{K_t}\right)^2-I_t\right]\right\} \tag{6}$$

I then derive the first-order condition of the return on investment as:

$$R^I_{t+1}=\frac{\left[\alpha\frac{Y_{t+1}}{K_{t+1}}+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right]+(1-\delta)\left[1+\phi\frac{I_{t+1}}{K_{t+1}}\right]}{1+\phi\left(\frac{I_t}{K_t}\right)} \tag{7}$$

given the concluding remarks of Cochrane (1991):

$$R^I_{t+1}=R^e_{t+1} \tag{8}$$

where $\frac{I_t}{K_t}$, $\frac{Y_t}{K_t}$ represent the q-factor factor and the profitability factor, respectively. Hou, Mo, Xue & Zhang (2020) decompose the investment return into two components—the dividend yield $\left[\alpha\frac{Y_{t+1}}{K_{t+1}}+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right]/\left[1+\phi\left(\frac{I_t}{K_t}\right)\right]$ and the capital gain $(1-\delta)\left[1+\phi\frac{I_{t+1}}{K_{t+1}}\right]/\left[1+\phi\left(\frac{I_t}{K_t}\right)\right]$. As mentioned above, I assume the production function as $Y_t=A_tf(K_t)$ and the total productivity factor process as $\log\frac{A_{t+1}}{A_t}=\Delta a_t=\nu_t+\sigma_t\epsilon_t, \epsilon_t\sim N(0,1)$. The component of dividend yield $\alpha\frac{Y_{t+1}}{K_{t+1}}$ is identical to the marginal production to capital:

$$\alpha\frac{Y_{t+1}}{K_{t+1}}=\alpha\frac{A_{t+1}K^{\alpha}_{t+1}}{K_{t+1}}=\alpha A_{t+1}K^{\alpha-1}_{t+1} \tag{9}$$

in which the capital accumulation process follows:

$$K_{t+1}=\left\{(1-\delta)K_t+\frac{\phi}{2}\left(\frac{I_t}{K_t}\right)^2K_t\right\}$$

Therefore, the profitability factor X_{t+1} can be viewed as the function of the q-factor and the productivity risks.

$$X_{t+1}\equiv\alpha\frac{Y_{t+1}}{K_{t+1}}=G\left(A_{t+1},\frac{I_t}{K_t}\right) \tag{10}$$

Equation (10) suggests that the productivity shock affects the return by the channel of marginal production to capital and dividend yields (i.e., the profitability factor) rather than the capital gain (i.e., the expected investment growth factor). This is the major difference between my work and Chen et al. (2020)'s work—I incorporate the TFP risks into the first-order conditional of productivity X_{t+1}, but Chen et al. (2020) introduce TFP risks into the expected growth factor $\frac{I_{t+1}/K_{t+1}}{I_t/K_t}$. As such, their work is more close to the investment-specific shock that contributes to capital accumulation rather than the first-order condition of productivity. Since Chen et al. (2020) and Hou, Mo, Xue & Zhang (2020) build the dynamic factor based on the investment-rate growth, my work can also be viewed as an alternative approach to build a dynamic factor model.

3 Empirical Methodology

The identification work consists of two components. First, I construct the productivity risk factors. To this end, I estimate the unconditional parameters and filtered probabilities of productivity risks at the beginning. I then consider the changes in beliefs on the mean and volatility states as risk factors. Second, I adopt Fama & Macbeth (1973) two-pass regression to investigate the pricing effects of such risk factors and test the performances of factor models that combine the TFP risk factors and Hou et al. (2015)'s q-factor model.

3.1 Data

1. Total Factor Productivity. I focus on quarterly TFP data from San Francisco Fed since 1963Q1 to 2020Q2 (Fernald 2012) [4]. I display the basic statistics in the Panel A of

[4] As we will consider the expected first-order difference of innovations, two period's values will be missed. As such, I consider two more quarters in the sample selection to keep it be consistent. Second, the portfolios are constructed from June 1963. Thus, the corresponding productivity risk factors had better be also formed by June 1963. To end, I skip the final two quarters to annualise the quarterly data. Another reason is, 1963

Table 1. Using macro data form Fed rather than Compustat is also the difference between my work and Chen et al. (2020)'s work [5].

2. Equity Portfolios. I consider annual return of 160 portfolios (i.e., 25 SIZE/BM portfolios, 25 SIZE/OP portfolios, 25 SIZE/INV portfolios, 25 BM/OP portfolios, 25 BM/INV portfolios, 25 OP/INV portfolios, 10 portfolios sorted on industry). Data are from 1967 to 2019, as I target the five-factor model. I adopt these collections of portfolios for three reasons. First, it is the cornerstone for numerous risk-factors widely accepted in the empirical asset pricing literature (Fama & French 1993, 2018, Hou et al. 2019). Second, it is an inclusive of acknowledged patterns in the cross-sectional stock returns. Besides, to avoid the critique of Lewellen et al. (2010), testing multifactor asset pricing models should base on various cross-sectional equity portfolios except for the 25 Size-Value portfolios [6]. I display the basic statistics in the Panel B of **Table 1**.

3. The q-Factor. I take data of q-factor model (e.g., the expected growth factor and the q-factor) during 1967 to 2019 from Professor Lu Zhang's website in which makes a factor model based on an equilibrium framework [7].

3.2 Identification Methodology

Estimating Productivity Dynamics Following Hamilton (1990)'s Expectation Maximisation algorithm, I estimate the conditional first-order moment $\nu_t \in \{\nu_h, \nu_l\}$ and second-order

is the beginning of Kennedy's tax cut. It would be better to capture the whole event of economic and tax reform.

[5]Details come from San Francisco Fed's website: https://www.frbsf.org/economic-research/indicators-data /total-factor-productivity-tfp/.

[6]Details are from Professor Ken French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french /data library.html.

[7]Details are in the following website: http://global-q.org/index.html.

moment $\sigma_t \in \{\sigma_h, \sigma_l\}$ of the productivity risks, as well as the transition probability matrices P^μ and P^σ. Since the Hidden Markov Chain process is unobservable, the agent's prior beliefs of the first and second-order moments of productivity growth captures the economic states. Namely. the prior beliefs equal prior probabilities. As such, the first principle of investment return and profitability are a function of the agent's beliefs. To this, end I define the prior beliefs that the conditional expectation and standard deviation are low in the next period as $b_{\nu,t}$ and $b_{\sigma,t}$, and I then compute such prior beliefs under the current information set F_t as:

$$\begin{aligned} b_{\nu,t} &= P(\nu_{t+1} = \nu_l | F_t) \\ b_{\sigma,t} &= P(\sigma_{t+1} = \sigma_l | F_t) \end{aligned} \tag{11}$$

Constructing Productivity Risk Factors The economic states are characterised by the product of the prior probabilities and the value conditional on the relevant state. Therefore, I specify the perceived mean and volatility of productivity growth as the weighted average based on prior probabilities:

$$\begin{aligned} \hat{\nu}_t &= b_{\nu,t}\nu_l + (1 - b_{\nu,t})\nu_h \\ \hat{\sigma}_t &= b_{\sigma,t}\sigma_l + (1 - b_{\sigma,t})\sigma_h \end{aligned} \tag{12}$$

Following Boguth & Kuehn (2013), I consider the first-order differences of both perceived moments as risk-factors:

$$\begin{aligned} \Delta\hat{\nu}_t &= \hat{\nu}_t - \hat{\nu}_{t-1} \\ \Delta\hat{\sigma}_t &= \hat{\sigma}_t - \hat{\sigma}_{t-1} \end{aligned} \tag{13}$$

To test the annual return data, all productivity risks factors above are annualised from the quarterly frequency.

Testing Portfolios by Risk Factors In this paper, I consider two models as the benchmark models to test portfolios. The first model is a four-factor model consisting of three TFP factors (i.e., the productivity growth, the conditional mean and volatility of productivity growth) and the q-factor, which can entirely yield from the first-order condition of investment return. The second model is a six-factor model that combines three productivity risk factors and the rest three factors (i.e., the q-factor, the market risk premiums factor, and the size factor) in Hou et al. (2015)'s q-factor model. I highlight the benchmark models as:

$$R^i_{t+1} - R^f_{t+1} = \begin{cases} \alpha^i_t + \beta_{TFP}\Delta TFP_t + \beta_\nu \Delta\hat{\nu}_t + \beta_\sigma \Delta\hat{\sigma}_t + \beta_q q_t + \mu^i_t \\ \alpha^i_t + \beta_{TFP}\Delta TFP_t + \beta_\nu \Delta\hat{\nu}_t + \beta_\sigma \Delta\hat{\sigma}_t + \beta_q q_t + \beta_{Mkt} Mkt_t + \beta_{SMB} SMB_t + \mu^i_t \end{cases} \tag{14}$$

I also test a large group of multifactor asset pricing models. The control group includes (i) Fama & French (1993, 2015, 2018) three-factor, five-factor, and six-factor model; (ii) the Hou et al. (2015) four-factor model and the Hou, Mo, Xue & Zhang (2020) five-factor model (i.e., the q^5 model); (iii) the Carhart (1997) four-factor model. I summarise these sets of asset pricing models as:

$$R^i_{t+1} - R^f_{t+1} = \begin{cases} \alpha^i_t + \beta_{Mkt} Mkt_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \mu^i_t \\ \alpha^i_t + \beta_{Mkt} Mkt_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{CMA} CMA_t + \beta_{RMW} RMW_t + \mu^i_t \\ \alpha^i_t + \beta_{Mkt} Mkt_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{CMA} CMA_t + \beta_{RMW} RMW_t + \beta_{MOM} MOM_t + \mu^i_t \\ \alpha^i_t + \beta_q q_t + \beta_{Mkt} Mkt_t + \beta_{SMB} SMB_t + \beta_{ROE} ROE_t + \mu^i_t \\ \alpha^i_t + \beta_q q_t + \beta_{EG} EG_t + \beta_{Mkt} Mkt_t + \beta_{SMB} SMB_t + \beta_{ROE} ROE_t + \mu^i_t \\ \alpha^i_t + \beta_{Mkt} Mkt_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{MOM} MOM_t + \mu^i_t \end{cases} \tag{15}$$

where the R^i_{t+1} and R^f_{t+1} denote the i th portfolio return and the risk-free rate, respectively. Mkt_t, SMB_t, HML_t, CMA_t, RMW_t, MOM_t represent the factors of market excess return, size, value, investment, profitability, and momentum from Fama & French (2018)'s six factor model. Unlike Chen et al. (2020), I do not incorporate the productivity risk factors into Fama & French (1993, 2015, 2018)'s multifactor pricing models in the control group, as their work bases on the inter-temporal CAPM model.

Two-Pass Regression Consider a k-factor asset pricing model, the vectors of risk loading, risk factors, and estimated risk premiums are given by $\boldsymbol{\beta}=\left(\beta_t^{i1},\beta_t^{i2},\ldots,\beta_t^{ik}\right)$, $\boldsymbol{F}=\left(F_t^1,F_t^2,\ldots,F_t^k\right)$ and $\boldsymbol{\lambda}=\left(\lambda^1,\lambda^2,\ldots,\lambda^k\right)$, respectively. The standard Fama & Macbeth (1973) regression $E_t[R_{t+1}^i-R_{t+1}^f]=\boldsymbol{\beta}\boldsymbol{\lambda}$ consists of two steps:

$$\begin{aligned} R_{t+1}^i-R_{t+1}^f &= const_1^{ik}+\beta_t^{ik}F_t^k+\varepsilon^{ik} \\ \bar{R}^i-\bar{R}^f &= const_2^k+\beta^{ik}\lambda^k+\varphi^k \end{aligned} \tag{16}$$

where the first step is time-series regression to yield the risk loadings $\hat{\boldsymbol{\beta}}=\left(\hat{\beta}_t^{i1},\hat{\beta}_t^{i2},\ldots,\hat{\beta}_t^{ik}\right)$, and the second step is cross-sectional regression to deliver the risk premiums $\hat{\boldsymbol{\lambda}}=\left(\hat{\lambda}^1,\hat{\lambda}^2,\ldots,\hat{\lambda}^k\right)$. $const_1^{ik}$ and $const_2^k$ are the constants in the first and second-pass regressions, respectively. ε^{ik} and φ^k report the residuals in both regression procedures.

Conditional Risk Premiums and Model Fitness Cochrane (2005) suggests that the price of excess return is identical to zero, which delivers:

$$E_t[M_{t,t+1}\left(R_{t+1}^i-R_{t+1}^f\right)]=0 \tag{17}$$

Therefore, the conditional risk premiums associate with the covariance between ex-post risk premiums and the stochastic discount factor, which are given by:

$$\begin{aligned} E_t[M_{t,t+1}\left(R_{t+1}^i-R_{t+1}^f\right)] &= E_t[M_{t,t+1}]E_t[\left(R_{t+1}^i-R_{t+1}^f\right)]+Cov_t(M_{t,t+1},R_{t+1}^i-R_{t+1}^f)=0 \\ \Rightarrow E_t[R_{t+1}^i-R_{t+1}^f] &= -\frac{Cov_t(R_{t+1}^i-R_{t+1}^f,M_{t,t+1})}{E_t[M_{t,t+1}]} \\ &= -\frac{Cov_t(R_{t+1}^i-R_{t+1}^f,M_{t,t+1})}{Var_t[M_{t,t+1}]}\times\frac{Var_t[M_{t,t+1}]}{E_t[M_{t,t+1}]} \end{aligned}$$

(18)

I follow Cochrane (1996) to identify the stochastic discount factor in an investment-based factor pricing model. Suppose a vector of stochastic discount factor of a k-factor asset pricing model to be $\boldsymbol{M_{t,t+1}} = \left(M^1_{t,t+1}, M^2_{t,t+1}, \ldots, M^k_{t,t+1}\right)$, in which $M^k_{t,t+1}$ is the stochastic discount factor of the k-th risk factor. As such, I derive the risk loadings as $\beta^{ik}_t = -\frac{Cov_t(R^i_{t+1}-R^f_{t+1}, M^k_{t,t+1})}{Var_t[M^k_{t,t+1}]}$ and risk premiums as $\lambda^k = \frac{Var_t[M^k_{t,t+1}]}{E_t[M^k_{t,t+1}]}$. I then compute the fitted cross-sectional risk premiums by:

$$E_t[R^i_{t+1} - R^f_{t+1}] = \hat{\beta}^{ik}_t \hat{\lambda}^k \tag{19}$$

from the results of Fama & Macbeth (1973) regression. My work measures the model fitness through comparing the fitted risk premiums $\hat{\beta}^{ik}_t \hat{\lambda}^k$ and the average sample portfolio excess return in a 45 degree line.

4 Empirical Results

I report several empirical results in this section. My report starts with the estimation of unconditional parameters, filtered probabilities, and the expected conditional mean and volatility of the aggregate productivity risks. Second, I present the constructed productivity risks factors in time-series level and report its relationship to other risk factors in Hou, Mo, Xue & Zhang (2020)'s q-factor model. Besides, I test the pricing performances of productivity risks factors and incorporate them into an investment-based asset pricing framework.

In my further asset pricing analysis, I estimate various factor pricing models adopting annual returns for 25 size/value, 25 size/profitability, 25 size/investment, 25 value/profitability, 25 value/investment, 25 profitability/investment, and 10 industry portfolios. The results of equally-weighted portfolios are reported from **Table 6** to **Table 10** with the corresponding expected risks premiums from **Figure 3** and **Figure 4**. Analogously, the results of value-weighted portfolios are presented from **Table 11** to **Table 15** with the corresponding fitted risks premiums from **Figure 5** and **Table 6**.

4.1 Productivity Process

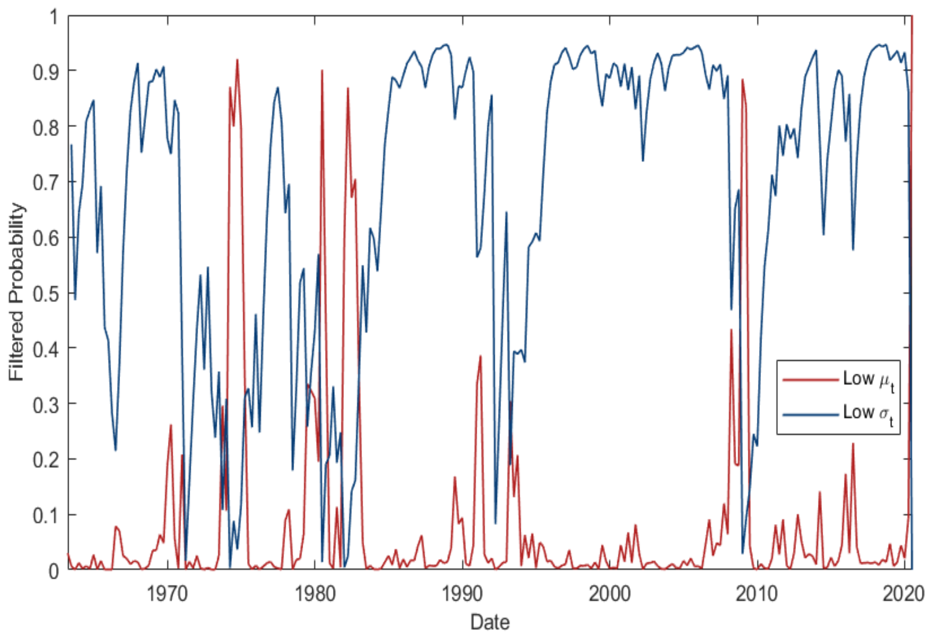
I present the estimated parameters of the Markov regime-switching model in **Table 2**. The quarterly expected growth rate is -0.9% in the low expected growth regime but would be 0.34% in the high expected growth regime, which the transition probabilities $p^{\mu}_{ll} = 0.7249$ and $p^{\mu}_{hh} = 1 - 0.0325 = 0.9675$, respectively. As such, the implied average duration of the recession state is 4 quarters while the expansion state is around 30 quarters, which is longer than the estimation of Hamilton (1990) and Liu & Miao (2015). I plot the filtered probabilities with the red line in **Figure 1**. The red line states that the prior probability spikes during the downturn, which implies a positive comovement between low expected economic growth and the business cycle recession.

Analogously, the quarterly expected volatility is 0.58% in the low regime but turns to be 1.11% in the high regime, whereas the transition probabilities $p^{\sigma}_{ll} = 0.9517$ and $p^{\sigma}_{hh} = 1 - 0.0902 = 0.9098$, respectively. As such, the implied average duration of the recession state is 11 quarters while the expansion state is around 20 quarters. In contrast to the sample from 1952 Boguth & Kuehn (2013), Liu & Miao (2015), my result suggests that the first-order moment of productivity risks is more persistent than the second-order moment. As such, my estimations suggest that the U.S economy expected more persistent expansions but less persistent recessions.

A possible explanation is, since the 1960's, Kennedy's supply-side tax cut policy made the 106 months expansions of the American economy. As such, the agent expects a longer duration of economic expansions. I plot the filtered probabilities of low expected volatility states with the blue line in **Figure 1**, which indicates that the prior probability spikes during the expansions. Therefore, the prior probability of low expected volatility positively

Figure 1: **Filtered Probabilities: Conditional Mean and Volatility**

Figure 1. This figure illustrates the estimated prior probabilities for the low expected growth state (red line) and low expected volatility state (blue line). I use Hamilton (1990)'s algorithm to estimate the results. Data cover 1963Q1 to 2020Q2.



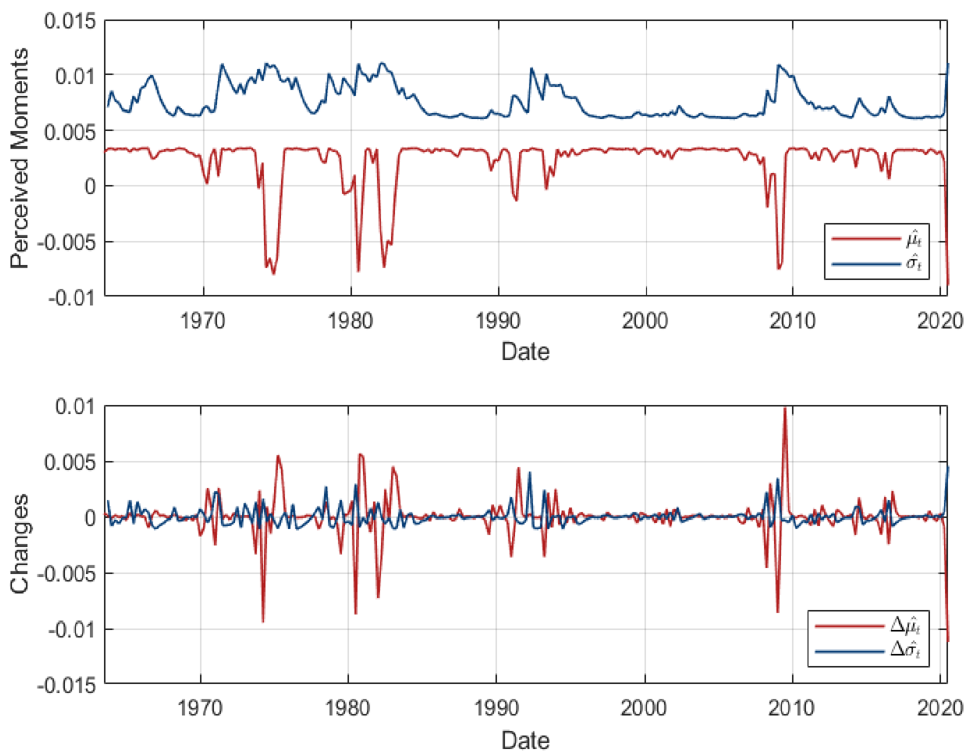
associated with the macroeconomic boom.

4.2 Production-Based Risk Factors

As suggested by Boguth & Kuehn (2013), the $AR(1)-GARCH(1,1)$ process plays a marginal role in capturing the variations of asset prices. Liu & Miao (2015) suggests that the $AR(1)-GARCH(1,1)$ productivity process only predicts 10% of the long horizon stock return in time-series. As such, I construct the risk factors based on the regime-switching model rather than the $AR(1)-GARCH(1,1)$ model and examine the cross-sectional implications of stock return. As mentioned above, I construct the risk factors by two steps. I first compute the perceived first and second-order moments by computing average states based on the prior probabilities. The results are reported in the first Panel of **Figure 2**. Second, I find the changes of belief innovations by the first-order differences of these perceived moments. I consider such changes in the belief innovations as productivity risk factors that are reported in the second panel of **Figure 2**.

Figure 2: **Risk Factors**

Figure 2 demonstrates the first and second-order perceived moments and their changes. I consider such changes in moments as risk factors as Boguth & Kuehn (2013). Data cover 1963Q1 to 2020Q2.



Further, I display the relation of productivity risk factors to other factors in the q-factor model. **Table 3** suggests that my three productivity growth risk factors Δa_t, $\Delta \nu_t$, and $\Delta \sigma_t$ have the coefficients of correlation -0.2195, -0.1834, and 0.2127 to the profitability factor.

In Chen et al. (2020)'s work, they report a correlation coefficient of macro uncertainty to expected growth factor as 0.22, but report an almost irrelevant relationship of aggregate uncertainty to profitability factor. Since they select the risk factor only by the statistical criteria, one cannot expect that they will introduce the TFP uncertainty risks into the ROE factor. As such, it would be not easy to incorporate their work into an equilibrium investment-based asset pricing model.

Second, I examine the performance of productivity risk factors using the standard Fama & Macbeth (1973) two-pass regression. First, for each portfolio in the cross-section, I adopt the time-series regression to estimate unconditional risk loadings of the portfolio excess returns on the productivity growth Δa_t, as well as the changes in perceived first and second-order moments of productivity growth $\Delta \nu_t$ and $\Delta \sigma_t$, respectively. Second, I use the cross-sectional regression of the expected excess return on risk-loadings mentioned above to obtain the prices of risks. This paper presents the result of the second procedure regression in **Table 4**. Finally, I consider the profitability factor as the control group in this case, as I treat the combination of productivity risks and q-factor as the proxy of first-order conditional of production to capital X_{t+1} while Hou et al. (2015) treat the ROE factor as the proxy.

4.3 From Risk Factors to Benchmark Models

In this section, I first report the results of productivity risk pricing, and then discuss how to construct the first benchmark investment-based q-Factor model starting with productivity risks. I also display another prospective to build my first benchmark model beginning with the q-Factor.

Using the R-Squared as a criterion, the estimation results from **Table 4** suggests that the productivity risks largely summarises the cross-sectional variation of equity return. In particular, the productivity uncertainty risks play a crucial role—it captures 67.1% of the stock return variation of 25 SIZE/BM portfolios and 40.2% of the equity return variation of

160 portfolios. Overall, the three productivity factors summarise around 85% of the stock return variation in 25 SIZE/BM portfolios and about 45% equity return variation of the 160 portfolios. Therefore, my estimation results suggest that three productivity growth factors are indeed priced risk factors. My work supports Boguth & Kuehn (2013)'s argument that macro uncertainty is a negatively priced source, but the role of the conditional mean of productivity growth is ambiguous. The reason is, the Newey & West (1987) T-statistics of the conditional mean of productivity growth are not always significant. As such, the estimator of the first-order moment of productivity risks may be indifferent from zero, and one cannot identify the role of the conditional mean of TFP risks in determining asset prices.

Different from Boguth & Kuehn (2013), the mere productivity risks cannot capture the full information in the stochastic discount factor—-one needs the risk factors from investment and capital in an equilibrium framework. As such, I propose the first benchmark model that consists of three productivity risk factors and q-factor as mentioned above from the first principle of investment.

Model IV in **Table 4** displays the performance of the first benchmark model. I show that my first benchmark model explains around 90% stock return variation in 25 SIZE/BM portfolios and about three-fourths equity return variation in 160 portfolios. As the first benchmark model are directly derived from the firm maximisation problem, my work suggests that a four-factor model entirely derived from the first-order condition of investment can largely summarise the equity return variation in equal-weighted portfolios.

As equation (10) suggest, both the profitability factor and the first benchmark model are proxies of the first-order condition of dividend yield. Thus, in **Table 4**, I also compare their explanation powers. Given that the T-statistic of single ROE factor is not significant different from zero in 95% confidence interval in the 25 SIZE/BM portfolios, I find that the first benchmark model outperforms the single profitability factor. Even though the estimator of the profitability factor is significant (-4.2819) in the case of 160 portfolio, the T-statistic of intercept is more significant (17.0617). As such, the first baseline model still outperforms the single ROE factor—the adjusted R^2 and RMSE can also confirm this argument. Overall, the first benchmark model could be an alternative candidate model that links the first order-condition to portfolio return in cross-section.

Table 4 offers the method to incorporate q-Factor into productivity risks, while **Table 5** provides an alternative perspective to construct the first benchmark model beginning with the q-Factor. Estimated results suggest that the single q-Factor can explain around 40% to 45% stock return variation in a wide set of cross-sectional portfolios. Adding the productivity uncertainty factor makes the model explains around 60% to 75% equity return variations that significantly improve the model performance. Second, adding all productivity risk factors would also improve the model performances and yield the first benchmark model.

The construction of the second benchmark model seems simple. To this end, I briefly incorporate all productivity risks into other factors (i.e., the market risk premiums factor, the size factor, the investment factor) of the q-factor model (Hou et al. 2015). As such, my work differs from Chen et al. (2020)'s work, as we build the benchmark models by economically explaining different risk factors—they explain the expected investment growth factor while I interpret the marginal production function.

4.4 Model Fitness

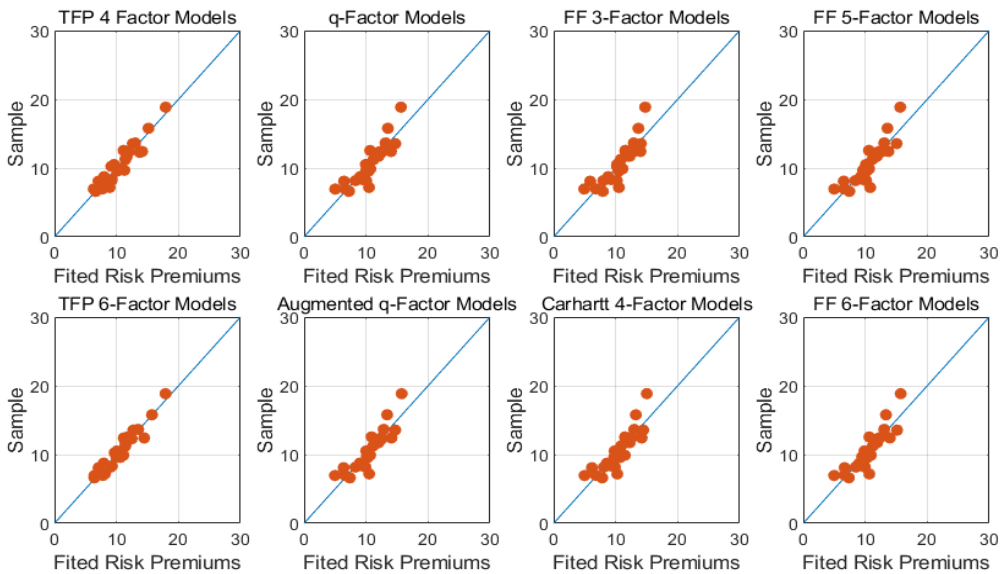
Table 6 reports the performances of various factor pricing models based on a partial equilibrium framework under 25 SIZE/BM equal-weighted portfolios. The Model I and Model VIII are my first and second benchmark model mentioned above. In this case, one can see the progress of how does the first benchmark model develop to the second benchmark model. Model IX and X are q-Factor models and q^5 model in Hou, Mo, Xue & Zhang (2020). I consider two criteria as the measurements of model performances—the adjusted R-Square and the root-mean-square error (RMSE).

Table 6 show that both benchmark models achieve a good fit in determining the standard 25 SIZE/BM equal-weighted portfolios, with about 90% in cross-sectional R-Squares and around 1 in the root-mean-square deviation. As such, the benchmark models significantly outperform the q-Factor model and q^5 model. Second, adding the expected growth factor into the productivity risks is ambiguous. The additional expected growth factor can sometimes improve the model performance but not a usual case, as the model V reports a higher value on the standard error (1.1 vs 1.08) and a lower value on F-tests (31.4 vs 41.2), R-Square (0.863 vs 0.870) in contrast to model IV and the degree of freedom. Second, the value of Newey & West (1987) T-statistics in the expected growth factor is too small. As such, the expected growth factor plays a marginal role in this case, and I do not incorporate the expected growth factor in my benchmark model at this stage.

Table 7 demonstrates that the performances of both benchmark model are comparable to the q-Factor model and q^5 model under the 160 portfolios. Surprisingly, the first benchmark model consisting of merely productivity risk factor and investment-rate factor has similar performance to many complicated multifactor models under broad sets of equity portfolios, as it interprets about three-fourths stock return variations in the equal-weighted 160 portfolios. As such, this paper argues that a parsimonious factor model purely derived from the first-order condition of investment return has comparable empirical fitness to multifactor models, providing a deeper economic background to understand the q-Factor series models.

Table 8 illustrates the testing results of Fama & French (1993, 2015, 2018) and Carhart (1997) factor pricing models under a broad range of equal-weighted stock portfolios. All multifactor models capture around 70% (0.688 to 0.700) equity return variations using the 25 SIZE/BM portfolios. However, under the 160 portfolios, the Fama & French (1993) three-factor model and Carhart (1997) four-factor model can merely explain around 60% equity return variations, which underperform other alternatives. Overall, using a wide range of equal-weighted equity portfolios, both benchmark models driven by productivity risks perform at least as good as Fama & French (2015, 2018) multifactor models and Hou et al. (2015), Hou, Mo, Xue & Zhang (2020) q-Factor series models in terms of the fitness. As such, the aggregate productivity can be one of the candidates of driving forces behind the investment-based q-Factor model.

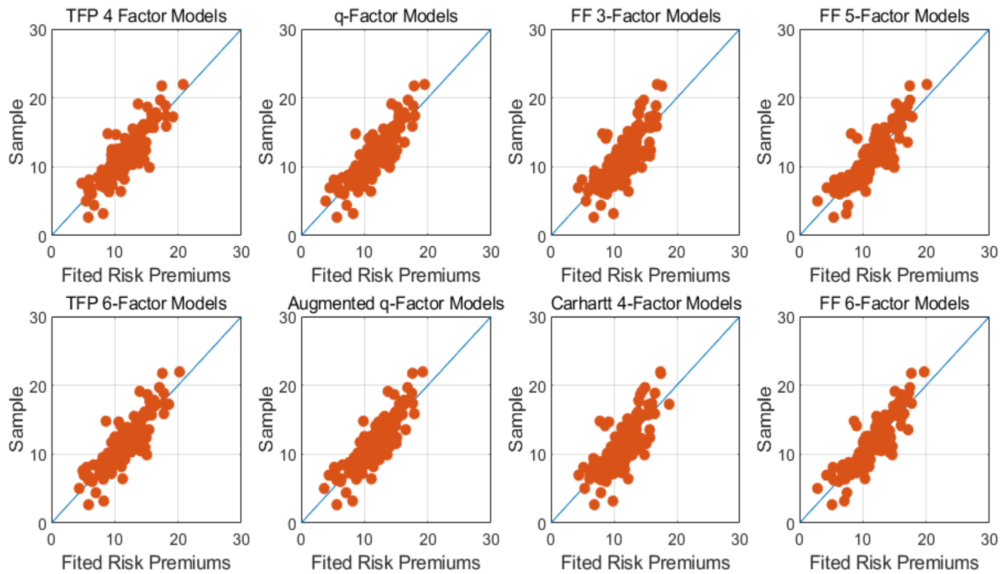
Figure 3: **Conditional Risk Premiums: 25 Equal-Weighted SIZE/BM Portfolios**



The empirical success of the productivity risks driving q-factor model is also demonstrated in **Figure 3** and **Figure 4**, in which I report the comparison between the fitted expected return and sample average return of the cross-sectional equity portfolios. **Figure 3** indicates that the fitted expected excess returns generated by both benchmark models (i.e., the TFP four-factor and six-factor model) are almost identical to sample observations, as the scatters nearly align in the 45-degree line. By contrast, the rest models (i.e., the Fama & French (1993, 2015, 2018) multifactor models, the Carhart (1997) four-factor model, and the Hou et al. (2015), Hou, Mo, Xue & Zhang (2020) q-Factor series models) in the control groups show significant deviations to the reference 45-degree line.

Figure 4 states the deviations of conditional risk premiums produced by all testing asset pricing models to their samples under the equal-weighted 160 portfolios. Except for the slightly larger deviations of Fama & French (1993) three-factor model and the Carhart

Figure 4: **Conditional Risk Premiums: 160 Equal-Weighted Portfolios**



(1997) four-factor model, the rest models predict similar deviations to their corresponding observations. The arguments from the goodness of fit and deviation are identical and consistent that the productivity risks driving q-Factor models show comparable performances to their multifactor peers.

Results in both **Table 4** and **Table 5** indicate that, by contrast to the productivity growth and its conditional mean, the time-varying productivity uncertainty plays the crucial role in explaining the stock return variations of cross-sectional equity portfolios. As such, I subtract the other two risk factors in both benchmark models and evaluate the performances based on the macroeconomic uncertainty driving q-Factor models. I report the test results in **Table 9** and **Table 10**. I denote Model I and Model VIII as the first and second degenerate benchmark models and address their performances, as they correspond to the first and second benchmark model above.

Given the R-Square as the criterion, results of **Table 9** and **Table 10** suggest that the first and second degenerate benchmark model can capture 74.8% and 77.5% equity return variations in the equal-weighted 25 SIZE/BM portfolios, which are also comparable to the q-Factor model and q^5 model, and slightly outperform the Fama-French and Carhart factor pricing models. Besides, the first and second degenerate benchmark model respectively predict 64.2% and 73.7% stock return variations in the equal-weighted 160 portfolios. As such, the first degenerate benchmark model still outperforms the Fama & French (1993) three-factor model (59.9%) and Carhart (1997) four-factor model (60.2%) even it underporforms the Fama & French (2015, 2018) five-factor (73.9%) and six-factor models (74.6%), as well as the q-Factor model (73.3%) and q^5 model (74.1%) . By contrast, the performance of second degenerate benchmark model is comparable to those multifactor models.

The tests of degenerate benchmark models state that productivity uncertainty serves as the most crucial driving force in all productivity risk factors. Therefore, one can adopt uncertainty driving factor models entirely derived from the first-order condition of investment return to capture most of the equity return variations in a wide range of equal-weighted cross-section stock portfolios.

4.5 Value Weighted Portfolios

In this section, I test the same models above using the 25 value-weighted SIZE/BM portfolios and 160 value-weighted portfolios. Empirical results in **Table 11** state that, the first and second benchmark models explain 71.9% and 75.6% of the stock return variations of 25 Size-Value portfolios. Despite the explanation powers of benchmark models drop, the fitness is at least as good as the q-Factor model (72.9%) and q^5 model (72.5%).

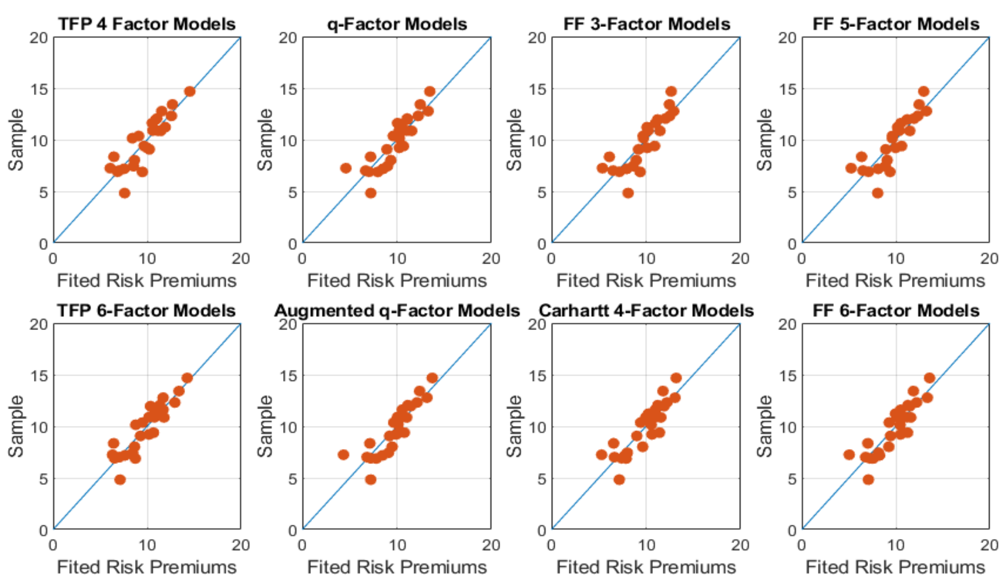
Further, under the 160 value-weighted portfolios, results from **Table 12** illustrate that the first benchmark model (Model I) merely explains half (48.2%) of the stock return variation of 160 portfolios (Model VIII), which faintly underperforms the q-Factor model (52.1%) and q^5 model (52.6%). By contrast, the second benchmark model interprets 60% equity return variation of 160 portfolios, which slightly outperforms the q-Factor model and

q^5 model.

What is more, I report the test results of control group asset pricing models in **Table 13**. The Fama & French (1993, 2015, 2018) three-factor, five-factor, and six-factor model captures 68%, 65.4%, and 71.9% stock return variation of 25 value-weighted SIZE/BM portfolios, respectively. Second, the Carhart (1997) four-factor model explains 73.1% equity return variation of the identical samples. Therefore, my first benchmark model is comparable to these multifactor asset pricing models and the second benchmark model faintly outperforms the above models.

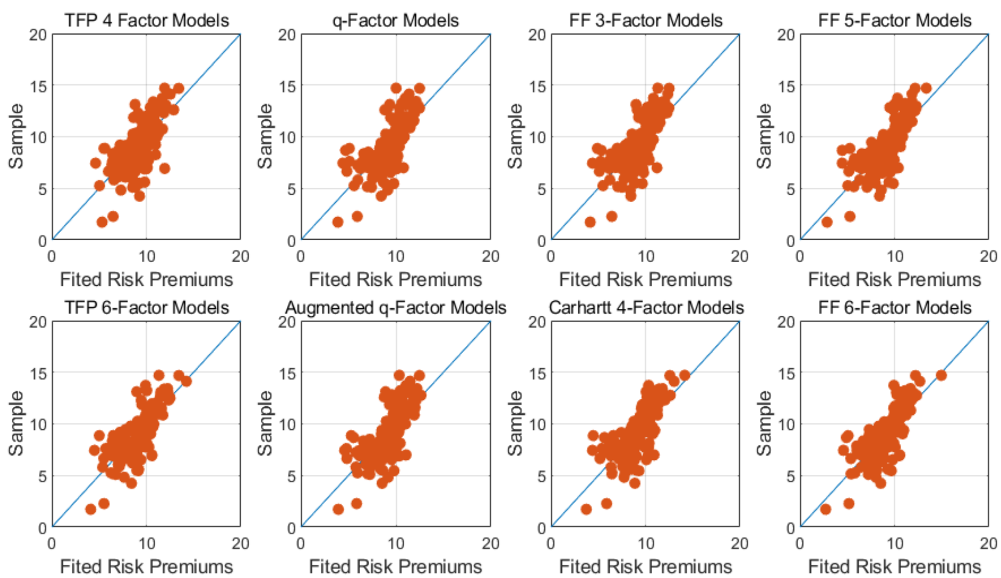
In the case of 160 value-weighted portfolios, most factor models merely explain about 50% of the cross-sectional stock return variations. Three models outperform their peers by capturing around 60% equity return variations, which are the FF five-factor model (58.4%), FF six-factor model (59.9%), and my second benchmark model (60%).

Figure 5: **Conditional Risk Premiums: 25 Value-Weighted Portfolios**



One can also be understand the empirical performances from **Figure 5** and **Figure 6**. The deviations of fitted expected excess return to sample average returns are larger than the case of equal-weighted portfolios. All models have similar performances in determining around 65% to 75% return variations on the 25 SIZE/Value portfolios, and about 50% to 60% variation of stock return on the 160 portfolios. As such, it would be not easy to distinguish from the figures. Despite the explanation powers decrease for all asset pricing models when testing value-weighted portfolios, the second benchmark model is still the leader in such a large group of factor models.

Figure 6: **Conditional Risk Premiums: 160 Value-Weighted Portfolios**



The discussion of value-weighted portfolios ends with the case of degenerate first and second benchmark model only driven by the macroeconomic uncertainty shock. The degenerate first benchmark model determines 61.9% and 37.5% of the return variations in the cases of 25 value-weighted SIZE/BM portfolios and 160 value-weighted portfolios, which underperforms most of the factor pricing models. By contrast, the degenerate first benchmark model captures 67.6% and 51% equity return variations of 160 cross-sectional portfolios, which is comparable to the q-Factor model and the q^5 model. Overall, the uncertainty driving q-factor model is comparable to other q-Factor series model considering a wide range of factor pricing models and equity portfolios.

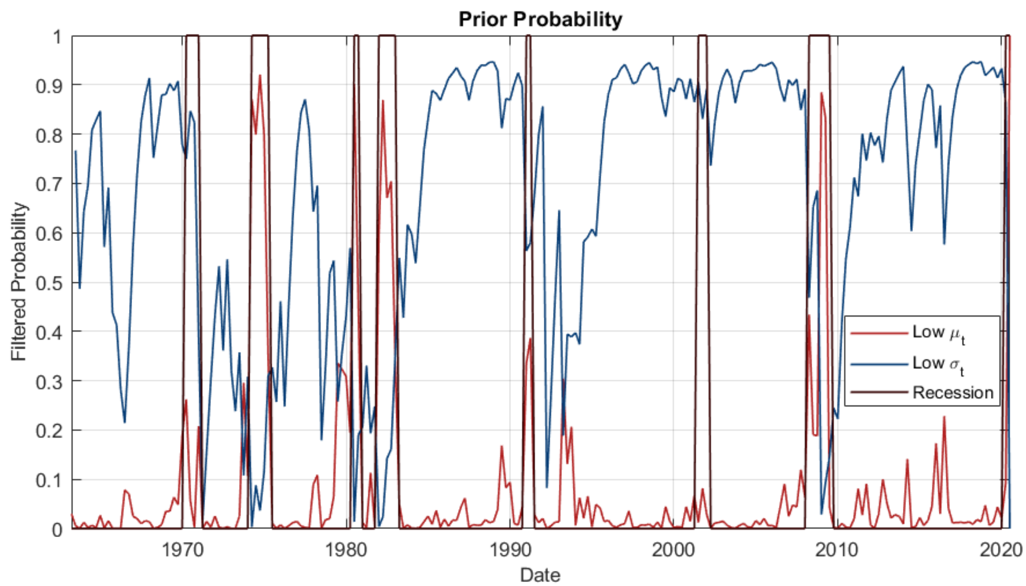
An additional finding is, under the 25 value-weighted SIZE/Value portfolios, the Newey & West (1987) T-statistics in the second benchmark model is not significant in the 95% confidence interval. However, this insignificance outlined above overturns in the case of 160 value-weighted portfolios. The degenerate second benchmark model has similar performances in all portfolios tests. As such, the first benchmark model directly derived from the first-order condition of investment is preferred in explaining the variation of return on 25 SIZE/Value portfolios. By contrast, the second benchmark model is preferred when I consider a large set of portfolios. The degenerate benchmark model can only be used to understand the mechanism but not to be applicable in the portfolio asset pricing.

4.6 Volatility States and Risk Premiums

A successful macro-asset pricing model should capture the relationship between economic recession and time-varying risk premiums. The literature of long-run risks suggests that the conditional mean of macroeconomic growth is procyclical but its conditional volatility is countercyclical such that it captures the bad economic states and produce higher expected risk premiums at such states.

For this reason, I display the relationship of NBER recession and filtered probabilities in **Figure 7**. The red, blue, and brown line represent the prior beliefs on the low expected growth states, the low expected volatility states, and the NBER recession indicator, respectively. The low expected growth states positively move with the recession indicator with a correlation coefficient 0.7324. In contrast, the low expected volatility states inversely move with the recession indicator with a correlation coefficient -0.4848. Therefore, the regime-switching model captures the macroeconomic fluctuations and successfully links to stock return variations.

Figure 7: **NBER Recession and Filtered Probability**



5 Conclusion

In this paper, I illustrate that the productivity risks, especially productivity volatility risks, can be a driving force behind the investment-based q-factor asset pricing model. To this end, I consider that the productivity risks as an unobservable four-state hidden Markov chains process and identify such a Markov process. I then construct productivity risk factors the regime-switching model and propose two benchmark investment-based q-factor models driven by productivity risks.

The tests results indicate that both benchmark models sufficiently summarise the equity return variation of broadly set cross-sectional portfolios. My results strongly support the long-run risks model, as I find that the macroeconomic uncertainty is an adversely priced source that mostly leads to risk premiums. The empirical success relies on the macroeconomic volatility can capture the states in the economic downturn. As such, productivity risks, particularly the productivity volatility, build a linkage between the business cycle fluctuations and financial markets.

The estimation results also provide insights to understand the q-Factor model and q^5 model. First, productivity risks connect the capital market and macroeconomics, as mentioned above. Second, like the q-Factor model and q^5 model, productivity risks can be derived from the first-order condition of investment return, particularly the marginal productivity function. Thus, aggregate productivity is one of the candidate economic forces behind the q-Factor model and q^5 model.

Table 1: **TFP Growth and 160 Portfolios: Summary Statistics**

Table 1 summarises the mean and standard deviation of the TFP growth and annual return in 160 portfolios in this study. I consider the 25 size/value, 25 size/profitability, 25 size/investment, 25 value/profitability, 25 value/investment, 25 profitability/investment, and 10 industry portfolios. Both of the equal-weighted and value-weighted cases are involved. The Productivity growth samples cover 1963Q1 to 2020Q2, and portfolio samples cover 1967 to 2019.

Panel A: Productivity Growth	
Average	Std.dev.
0.8711	3.2532

Panel B: Cross-Sectional Portfolios					
Characteristics	Portfolios	Equal-Weighted		Value-Weighted	
		Average	Std.dev.	Average	Std.dev.
25 SIZE/BM	SIZE1/BM5	23.5861	33.1606	19.6173	27.8412
	SIZE5/BM1	11.6839	19.7774	11.7231	18.8951
25 SIZE/OP	SIZE1/OP1	18.3780	38.5377	12.7476	32.3342
	SIZE5/OP5	13.0864	18.7728	12.5644	18.1049
25 SIZE/INV	SIZE1/INV1	24.4257	42.2256	17.8274	32.9988
	SIZE5/INV5	10.0844	22.8453	11.5719	22.4172
25 BM/OP	BM1/OP1	9.7455	40.6295	7.1760	32.8853
	BM5/OP5	22.9081	52.4478	19.6992	34.6995
25 BM/INV	BM1/INV1	17.1434	40.9808	12.3118	22.5537
	BM5/INV5	16.9318	33.8415	12.2487	25.6547
25 OP/INV	OP1/INV1	22.6249	40.4320	12.5644	18.1049
	OP5/INV5	13.7537	28.9963	12.9890	18.9816
10 Industry	Industry 1	14.9166	26.4438	13.5916	17.4063
	Industry 10	16.2909	26.8795	12.1743	20.1529

Table 2: **Markov Model of Productivity Growth**

This table illustrates estimated parameters of the Markov model for log productivity growth Δa_t: $\log \frac{A_{t+1}}{A_t} = \Delta a_t = \nu_t + \sigma_t \epsilon_t, \epsilon_t \sim N(0,1)$, in which the conditional first-order moment $\nu_t \in \{\nu_h, \nu_l\}$ and second-order moment $\sigma_t \in \{\sigma_h, \sigma_l\}$. The conditional expectation and standard deviation processes change with transition matrices P^μ and P^σ, which are denoted by

$$P^\mu = \begin{bmatrix} p_{ll}^\mu & 1 - p_{ll}^\mu \\ 1 - p_{hh}^\mu & p_{hh}^\mu \end{bmatrix}, P^\sigma = \begin{bmatrix} p_{ll}^\sigma & 1 - p_{ll}^\sigma \\ 1 - p_{hh}^\sigma & p_{hh}^\sigma \end{bmatrix}.$$

My algorithm follows Hamilton (1990). I adopt quarterly productivity growth for the years 1963Q1 to 2020Q2. Standard errors are reported in parentheses.

Parameters	ν_l	ν_h	σ_l	σ_h	p_{ll}^μ	$1 - p_{hh}^\mu$	p_{ll}^σ	$1 - p_{hh}^\sigma$
Values	-.0090	.0034	.0058	.0111	.7249	.0325	.9517	.09015
std.err	(.0035)	(.0007)	(.0010)	(.0020)	(.1499)	(.0200)	(.0305)	(.0999)

Table 3: **Factor Correlations**

This table reports the correlation of productivity risks factors to other factors in the q-factor model. I display three productivity factors Δa_t, $\Delta \nu_t$, and $\Delta \sigma_t$, as well as the q-factor, expected growth factor, profitability factor, market risk factor, and size factor in the augmented q-factor model Hou, Mo, Xue & Zhang (2020).

Panel A: Factor Correlation

Variables	Δa_t	$\Delta \nu_t$	$\Delta \sigma_t$	Q	EG	Mkt	SMB	ROE
Δa_t	1.0000	0.5269	-0.0789	-0.0500	-0.0231	0.5136	0.1643	-0.2195
$\Delta \nu_t$		1.0000	-0.3747	0.1210	0.0764	0.3628	0.1980	-0.1834
$\Delta \sigma_t$			1.0000	-0.1109	-0.0275	-0.2056	-0.2061	0.2127
Q				1.0000	0.3127	-0.2754	0.1080	-0.1377
EG					1.0000	-0.4377	-0.2623	0.3623
Mkt						1.0000	0.1830	-0.2452
SMB							1.0000	-0.2034
ROE								1.0000

Table 4: **Productivity Risk Pricing**

This table presents the estimated market prices of risks from the second step cross-sectional regression. I consider three productivity risk factors Δa_t, $\Delta \nu_t$, and $\Delta \sigma_t$, as well as the q-factor and the profitability factor. In this table, I construct the first benchmark model (Model IV) beginning with the productivity risk factor. Panel A and B reports the results of 25 Size-Value Portfolios and 160 portfolios, respectively. I report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data cover 1967 to 2019 with annual frequency.

Models	Intercept (t-stat)	Δa_t (t-stat)	$\Delta \nu_t$ (t-stat)	$\Delta \sigma_t$ (t-stat)	Q	ROE	Adj-R^2	RMSE	F-Test	P-Value	DOF
Panel A: 25 Value-Size Portfolios											
I	4.8169	3.7338					0.162	2.74	5.64	0.0263	23
	(1.6746)	(1.8868)									
II	6.5017		0.0016				0.077	2.87	3	0.0968	23
	(2.6440)		(1.5899)								
III	1.7230			0.0019			0.671	1.71	50	3.34e-07	23
	(1.1678)			(-5.9327)							
IV	1.0506	4.0838	-0.0014	-0.0014			0.827	1.24	39.4	8.38e-09	21
	(1.2247)	(5.6560)	(-2.6936)	(-5.8865)							
V	1.9768	4.8137	-0.0004	-0.0009	1.4270		0.870	1.08	41.2	2.17e-09	20
	(2.3676)	(8.7774)	(-0.7972)	(-4.3798)	(2.4267)						
VI	6.8443					-5.1303	0.171	2.72	5.95	0.0228	23
	(3.8943)					(-1.9492)					
Panel B: 160 Portfolios											
I	4.8318	3.9944					0.175	3.12	34.7	1.12e-21	158
	(4.8318)	(3.9944)									
II	6.8645		0.0016				0.133	3.20	25.3	1.31e-06	158
	(8.1394)		(5.0869)								
III	4.5456			-0.0014			0.402	2.66	20.7	1.32e-19	158
	(4.6337)			(-7.1109)							
IV	3.7673	1.8138	0.0004	-0.0013			0.433	2.59	41.5	9.2e-20	158
	(3.6774)	(1.6066)	(0.7683)	(-5.0408)							
V	4.5238	4.7521	0.0015	-0.0003	5.3547		0.736	1.76	112	1.15e-44	155
	(6.4193)	(8.7671)	(4.5397)	(-2.0672)	(9.4756)						
IV	9.0023					-3.0434	0.110	3.24	20.7	0.0228	158
	(17.0617)					(-4.2819)					

Table 5: **The q-Factor Risk Pricing**

This table presents the estimated market prices of risks from the second step cross-sectional regression. I consider three productivity risk factors Δa_t, $\Delta \nu_t$, and $\Delta \sigma_t$, as well as the q-factor. In this table, I construct the first benchmark model (Model III) starting with the q-factor. Panel A and B reports the results of 25 Size-Value Portfolios and 160 portfolios, respectively. I report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data cover 1967 to 2019 with annual frequency.

Models	Intercept (t-stat)	Δa_t (t-stat)	$\Delta \nu_t$ (t-stat)	$\Delta \sigma_t$ (t-stat)	Q	Adj-R^2	RMSE	F-Test	P-Value	DOF
				Panel A: 25 Value-Size Portfolios						
I	11.8300				3.7323	0.400	2.31	17	0.000411	23
	(12.5254)				(5.5130)					
II	4.0391			-0.0016	2.9709	0.748	1.5	36.6	1e-07	22
	(2.7185)			(-4.6584)	(6.4066)					
III	1.9768	4.8137	-0.0004	-0.0009	1.4270	0.870	1.08	41.2	2.17e-09	20
	(2.3676)	(8.7774)	(-0.7972)	(-4.3798)	(2.4267)					
				Panel B: 160 Portfolios						
I	13.1230				5.1096	0.437	2.58	124	1.12e-21	158
	(31.3766)				(7.7236)					
II	7.5980			-0.0011	4.6631	0.637	2.07	141	1.12e-21	157
	(10.0960)			(-8.4937)	(9.2123)					
III	4.5238	4.7521	0.0015	-0.0003	5.3547	0.736	1.76	112	1.15e-44	155
	(6.4193)	(8.7671)	(4.5397)	(-2.0672)	(9.4756)					

Table 6: **Productivity Risks in ICAPM: 25 Equal-Weighted SIZE/BM Portfolios**

Table 6 reports the results of cross-sectional regression of 25 equal-weighted SIZE/BM portfolios on a large set of factor pricing models built on investment-based framework. I test both benchmark models and report the progress about how does the first benchmark model develop to the second benchmark model. I also report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. The q-Factor model and q^5 model are in the control group. Data cover 1967 to 2019.

Models	I	II	III	IV	V	VI	VII	VIII	IX	X
Cons.	1.9768	1.0964	5.6081	2.2332	1.9434	6.6915	6.2710	6.4849	9.2458	9.0449
(t-stat)	(2.3676)	(1.6766)	(7.3131)	(1.2875)	(1.9678)	(6.8789)	(6.9344)	(6.3973)	(2.9844)	(2.8826)
Δa_t	4.8137	6.1227	5.8405	3.1563	4.9181	5.0743	5.3435	5.3482		
(t-stat)	(8.7774)	(7.2650)	(13.2872)	(2.4083)	(4.2483)	(6.5543)	(6.6214)	(6.2507)		
$\Delta \nu_t$	-0.0004	-0.0002	0.0003	-0.0018	0.0004	-0.0001	0.0002	0.0000		
(t-stat)	(-0.7972)	(-0.2479)	(0.8549)	(-3.0791)	(-0.6500)	(0.0258)	(0.4729)	(0.0893)		
$\Delta \sigma_t$	-0.0009	-0.0014	-0.0011	-0.0013	-0.0009	-0.0011	-0.0011	-0.0011		
(t-stat)	(-4.3798)	(-7.8259)	(-8.6722)	(-4.4787)	(-2.9221)	(-3.8908)	(-3.6834)	(-3.9434)		
Q	1.4270				1.4000	0.7643	0.9872	0.5927	4.1526	4.2722
(t-stat)	(2.4267)				(2.0408)	(1.1450)	(1.3960)	(0.8942)	(4.8092)	(5.1716)
EG		1.7011			0.0840	0.3858	-0.5369			-0.7579
(t-stat)		(1.6382)			(0.0585)	(0.4247)	(-0.4129)			(-0.3106)
Mkt			2.2425			1.0130	1.3787	1.3231	-2.0446	-1.9168
(t-stat)			(3.1675)			(1.1312)	(1.7412)	(1.4239)	(-0.6578)	(-0.6139)
SMB				3.0899			2.5526	2.7084	3.5266	3.5020
(t-stat)				(6.1162)			(12.4733)	(11.2249)	(5.4311)	(5.2817)
ROE									-2.8019	-3.4290
(t-stat)									(-1.3313)	(-1.3836)
Ajd-R^2	0.870	0.851	0.905	0.823	0.863	0.899	0.895	0.896	0.732	0.720
RMSE	1.08	1.15	0.921	1.26	1.1	0.948	0.926	0.962	1.55	1.58
F-test	41.2	35.2	58.2	29	31.4	36.8	30.3	35.6	17.4	13.3
P-value	2.17e-09	8.7e-09	9.85e-11	4.55e-08	1.5e-08	3.78e-09	2.11e-08	4.91e-09	2.71e-06	7.87e-10
DOF	20	20	20	20	19	18	17	18	20	19

Table 7: **Productivity Risks in ICAPM: 160 Equal-Weighted Portfolios**

Table 7 reports the results of the second step regression of 160 equal-weighted portfolios on a large set of factor pricing model built on investment-based framework. I test both benchmark models and report the progress about how does the first benchmark model develop to the second benchmark model. I also report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. The q-Factor model and q^5 model are in the control group. Data cover 1967 to 2019.

Models	I	II	III	IV	V	VI	VII	VIII	IX	X
Cons.	4.5238	4.1643	9.4351	5.1989	4.5237	4.8469	6.2572	6.1168	3.0400	4.5312
(t-stat)	(6.4193)	(4.2912)	(8.2375)	(3.9460)	(6.4111)	(2.6825)	(5.0597)	(4.2029)	(1.8166)	(2.9965)
Δa_t	4.7521	7.3815	6.0386	-0.1311	4.7904	4.8334	3.3853	3.3369		
(t-stat)	(8.7671)	(7.3625)	(7.1554)	(-0.0962)	(5.6732)	(6.4992)	(4.3978)	(4.3934)		
$\Delta \nu_t$	0.0015	0.0021	0.0021	0.0002	0.0015	0.0015	0.0016	0.0014		
(t-stat)	(4.5397)	(3.3460)	(3.3985)	(0.4330)	(3.8475)	(3.8290)	(5.0844)	(3.7653)		
$\Delta \sigma_t$	-0.0003	-0.0011	-0.0008	-0.0013	-0.0003	-0.0003	-0.0004	-0.0003		
(t-stat)	(-2.0672)	(-5.3135)	(-4.4483)	(-5.0655)	(-1.9816)	(-1.9388)	(-2.4806)	(-2.0208)		
Q	5.3547				5.3442	5.2744	5.3741	5.4349	6.4056	6.2331
(t-stat)	(9.4756)				(8.4202)	(9.3775)	(9.2395)	(9.4770)	(11.7382)	(10.4274)
EG		6.1221			1.6396	1.6344	3.9271			3.2631
(t-stat)		(6.4685)			(1.8513)	(1.7827)	(2.4317)			(1.9619)
Mkt			-1.4888			2.7609	0.8548	0.9469	3.8325	2.4234
(t-stat)			(-1.1497)			(1.2874)	(0.6426)	(0.6042)	(2.1728)	(1.6278)
SMB				3.7853			3.6051	3.3721	4.0404	4.1150
(t-stat)				(8.6893)			(8.5745)	(9.3726)	(11.2495)	(12.6913)
ROE									-1.2958	-0.5447
(t-stat)									(-2.0976)	(-0.7282)
Adj-R^2	0.736	0.596	0.635	0.443	0.734	0.733	0.750	0.742	0.733	0.741
RMSE	1.76	2.18	2.07	2.56	1.77	1.74	1.72	1.74	1.77	1.75
F-test	112	59.7	70.2	32.7	89.1	73.7	69.1	77.4	110	92
P-Value	1.15e-44	1.96e-30	8.02e-34	9.58e-20	1.21e-43	1.2e-42	4.85e-44	7.91e-44	2.58e-44	1.92e-44
DOF	155	155	155	155	154	153	152	153	155	154

Table 8: **Equal-Weighted Portfolios: Fama-French & Carhartt Factor Models**

Table 8 evaluates the performances of many widely accepted asset pricing models under both the equal-weighted 25 SIZE/BM portfolios and 160 equal-weighted portfolios, respectively. I test (i) Fama & French (1993, 2015, 2018) three-factor, five-factor, and six-factor model; (ii) the Carhart (1997) four-factor model in the control group. I then report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data are from 1967 to 2019.

Portfolios	Panel A: 25 Value-Size Portfolios				Panel B: 160 Portfolios			
Models	FF3	Carhart 4	FF5	FF6	FF3	Carhart 4	FF5	FF6
Cons.	7.8624	9.5051	9.9797	12.1135	2.2169	4.3206	2.0807	-1.3109
(t-stat)	(3.3119)	(3.5203)	(2.5533)	(2.6808)	(1.1244)	(1.4361)	(1.4731)	(-0.5971)
Mkt	-0.9008	-3.3034	-2.6857	-5.1467	4.6051	2.2951	3.8817	7.3827
(t-stat)	(-0.3857)	(-1.1086)	(-0.7596)	(-1.1301)	(2.2717)	(0.7000)	(2.8702)	(3.1639)
SMB	3.7570	3.6280	3.3132	3.2060	3.8482	3.8518	4.0924	4.1463
(t-stat)	(4.6000)	(4.5711)	(5.9314)	(5.4155)	(9.2210)	(9.2073)	(11.6695)	(13.0347)
HML	5.4594	5.3651	5.4353	5.4407	7.8111	7.4906	6.8888	7.1248
(t-stat)	(4.4675)	(4.5524)	(5.0939)	(5.1039)	(8.2731)	(8.5094)	(9.7060)	(9.9364)
CMA			5.6280	5.5467			7.6361	7.8433
(t-stat)			(2.9340)	(2.9436)			(11.7101)	(11.7167)
RWM			-2.4457	-2.5049			-0.4325	-0.4513
(t-stat)			(-0.8232)	(-0.8804)			(-0.5684)	(-0.6039)
MOM		-7.8968		-5.3221		-5.3693		-3.0327
(t-stat)		(-2.0419)		(-1.2507)		(-3.8358)		(-2.3438)
Adj-R^2	0.699	0.698	0.700	0.688	0.599	0.602	0.739	0.746
RMSE	1.64	1.63	1.64	1.67	2.18	2.17	1.75	1.73
F-test	19.6	14.9	12.2	9.82	80.1	69.1	91.1	78.7
P-Value	2.71e-06	8.61e-06	2.15e-05	7.19e-05	1.99e-31	6.33e-31	3.36e-44	3.1e-44
DOF	21	20	19	18	156	155	154	153

Table 9: **Volatility Risks in ICAPM: 25 Equal-Weighted Size/BM Portfolios**

Table 9 reports the results of cross-sectional regression of 25 equal-weighted SIZE/BM portfolios on various investment-based factor pricing models only driven by productivity uncertainty risks. I then report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data are from 1967 to 2019.

Models	I	II	III	IV	V	VI	VII	VIII
Cons.	4.0391	2.9532	4.9169	1.7208	4.0515	5.8418	7.2102	7.3868
(t-stat)	(2.7185)	(2.2676)	(3.1707)	(1.0244)	(2.9342)	(3.0103)	(3.3063)	(3.3652)
$\Delta\sigma_t$	-0.0016	-0.0020	-0.0020	-0.0019	-0.0009	-0.0010	-0.0010	-0.0010
(t-stat)	(-4.6584)	(-5.9373)	(-5.8259)	(-3.6730)	(-1.7359)	(-1.8492)	(-2.0156)	(-2.1685)
Q	2.9709				3.8616	3.5567	3.2693	3.1622
(t-stat)	(6.4066)				(7.2089)	(6.9498)	(5.7741)	(6.3721)
EG		1.6487			-1.5212	-2.1888	-0.0602	
(t-stat)		(3.0130)			(-1.0206)	(-1.7976)	(-0.0330)	
Mkt			2.3881			0.9816	-0.3217	-0.4368
(t-stat)			(2.1457)			(0.5559)	(-0.1545)	(-0.2063)
SMB				3.8018			4.0403	4.0630
(t-stat)				(6.5240)			(5.5788)	(5.7588)
Adj-R^2	0.748	0.689	0.711	0.656	0.774	0.769	0.765	0.775
RMSE	1.5	1.67	1.6	1.75	1.42	1.43	1.45	1.42
F-test	36.6	27.6	30.6	29	28.4	21	16.6	21.7
P-value	1e-07	1e-06	4.43e-07	3.04e-06	1.4e-07	6.22e-07	2.33e-06	4.83e-07
DOF	22	22	22	22	21	20	19	20

Table 10: **Volatility Risks in ICAPM: 160 Equal-Weighted Portfolios**

Table 10 displays the results of second-pass regression of 160 equal-weighted stock portfolios on various investment-based factor pricing models only driven by productivity uncertainty risks. I then report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data are from 1967 to 2019.

Models	I	II	III	IV	V	VI	VII	VIII
Cons.	7.5980	7.0225	8.6590	4.3376	6.2373	3.8498	6.1819	5.7948
(t-stat)	(10.0960)	(6.4212)	(7.6497)	(5.7419)	(10.4660)	(2.9410)	(5.9777)	(4.5136)
$\Delta\sigma_t$	-0.0011	-0.0017	-0.0016	-0.0012	-0.0005	-0.0004	-0.0005	-0.0004
(t-stat)	(-8.4937)	(-7.1178)	(-7.1002)	(-4.3838)	(-2.9580)	(-3.0280)	(-3.9975)	(-2.6614)
Q	4.6631				5.5065	6.2095	5.7053	5.8744
(t-stat)	(9.2123)				(8.6614)	(12.5847)	(10.5194)	(11.5018)
EG		3.0847			-1.5857	-0.3690	4.0131	
(t-stat)		(4.2976)			(-1.9493)	(-0.3378)	(2.6546)	
Mkt			-0.0632			3.6738	0.7577	1.0468
(t-stat)			(-0.0633)			(2.3945)	(0.6999)	(0.7533)
SMB				3.3440			4.2161	4.1017
(t-stat)				(8.2599)			(12.8554)	(11.2347)
Adj-R^2	0.637	0.477	0.478	0.410	0.679	0.696	0.745	0.730
RMSE	2.07	2.48	2.48	2.64	1.95	1.89	1.74	1.78
F-test	141	59.7	73.8	56.3	113	91.9	93.7	109
P-Value	9.63e-36	3.15e-23	2.49e-23	3.61e-19	6.3e-39	7.03e-40	6.66e-45	5.98e-44
DOF	157	157	157	157	156	155	154	155

Table 11: **Productivity Risks in ICAPM: 25 Value-Weighted SIZE/BM Portfolios**

Table 11 reports the results of cross-sectional regression of 25 value-weighted SIZE/BM portfolios on a large group of factor pricing models built on investment-based framework. I test both benchmark models and the q-Factor model and q^5 model are in the control group. I also report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data cover 1967 to 2019.

Models	I	II	III	IV	V	VI	VII	VIII	IX	X
Cons.	3.8785	2.8327	6.6965	4.3462	4.5902	9.3046	8.7488	9.7843	16.2660	15.0010
(t-stat)	(2.1790)	(1.5743)	(3.7423)	(1.8049)	(2.7554)	(6.0120)	(5.8952)	(6.2127)	(4.0752)	(4.2470)
Δa_t	3.9177	4.7298	4.8214	1.9542	2.6808	3.0771	3.2828	3.4139		
(t-stat)	(4.7633)	(3.8261)	(7.2049)	(1.0052)	(2.0090)	(2.2848)	(2.2726)	(2.6671)		
$\Delta \nu_t$	-0.0003	-0.0007	-0.0002	-0.0020	-0.0006	-0.0008	-0.0007	-0.0008		
(t-stat)	(-0.4787)	(-0.7120)	(-0.4046)	(-2.1079)	(-1.1154)	(-1.5523)	(-1.4818)	(-1.7893)		
$\Delta \sigma_t$	-0.0006	-0.0011	-0.0009	-0.0010	-0.0003	-0.0006	-0.0006	-0.0008		
(t-stat)	(-2.5636)	(-5.8436)	(-5.9513)	(-3.3249)	(-1.4304)	(-1.6177)	(-1.6738)	(-1.8445)		
Q	1.5888				2.0422	0.4558	0.5465	0.0424	2.7808	2.8758
(t-stat)	(2.3156)				(2.4883)	(0.3388)	(0.4075)	(0.0328)	(6.2387)	(6.4372)
EG		1.4118			-1.4947	-1.2227	-1.7433			-2.1531
(t-stat)		(1.4279)			(-1.3376)	(-1.0608)	(-1.5088)			(-1.2026)
Mkt			1.3449			-1.4608	-0.8566	-1.9672	-8.3039	-7.0678
(t-stat)			(0.8669)			(-0.9102)	(-0.5545)	(-1.1884)	(-2.0913)	(-2.0086)
SMB				1.9850			2.1028	2.1973	2.7344	2.6715
(t-stat)				(2.3598)			(4.7374)	(4.4256)	(5.8991)	(6.1075)
ROE									-5.9071	-6.4855
(t-stat)									(-2.0750)	(-2.2060)
Adj-R^2	0.719	0.665	0.764	0.627	0.715	0.772	0.759	0.756	0.729	0.725
RMSE	1.31	1.15	1.2	1.5	1.31	1.18	1.21	1.22	1.28	1.29
F-test	16.4	12.9	20.4	11.1	13.1	14.5	11.8	13.4	17.1	13.7
P-value	4.28e-06	2.39e-05	7.77e-07	6.7e-05	1.34e-05	4.97e-06	1.96e-05	8.93e-06	3.03e-06	9.77e-06
DOF	20	20	20	20	19	18	17	18	20	19

Table 12: **Productivity Risks in ICAPM: 160 Value-Weighted SIZE/BM Portfolios**

Table 12 displays the results of the second step regression of 160 value-weighted portfolios on a large group of factor pricing models built on investment-based framework. I test both benchmark models and the q-Factor model and q^5 model are in the control group. I also report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data cover 1967 to 2019.

Models	I	II	III	IV	V	VI	VII	VIII	IX	X
Cons.	5.1217	4.9709	9.4809	8.1302	5.0605	9.7845	10.6920	10.7810	9.1063	9.5088
(t-stat)	(8.1584)	(6.3814)	(14.9083)	(7.3708)	(8.1538)	(9.3890)	(9.8211)	(9.7086)	(6.0352)	(6.2829)
Δa_t	3.4649	3.9569	4.4192	-0.8423	3.6980	3.7143	2.7931	2.8231		
(t-stat)	(6.7240)	(5.5939)	(7.5746)	(-0.8335)	(6.5263)	(7.5821)	(4.4406)	(4.6575)		
$\Delta \nu_t$	0.0006	0.0012	0.0015	-0.0007	0.0007	0.0010	0.0009	0.0009		
(t-stat)	(1.6883)	(2.9969)	(3.8268)	(-1.9271)	(1.9759)	(2.9849)	(3.0979)	(2.5048)		
$\Delta \sigma_t$	-0.0003	-0.0009	-0.0007	-0.0004	-0.0003	-0.0005	-0.0004	-0.0004		
(t-stat)	(-2.1632)	(-7.3481)	(-6.5704)	(-2.0772)	(-2.2207)	(-3.7634)	(-3.4172)	(-3.0458)		
Q	2.5862				2.5984	1.8557	1.8670	1.8182	2.2357	2.3183
(t-stat)	(6.8073)				(6.9767)	(4.7814)	(5.3888)	(4.5670)	(5.4612)	(6.0860)
EG		3.4549			1.0807	0.8106	2.6087			2.4111
(t-stat)		(5.9354)			(1.7576)	(1.6676)	(2.9749)			(2.3030)
Mkt			-1.5409			-1.9016	-3.0774	-3.1199	-1.5952	-2.0490
(t-stat)			(-2.3870)			(-1.7797)	(-2.7164)	(-2.7244)	(-1.0338)	(-1.3217)
SMB				2.3819			2.5942	2.5126	2.7573	2.8969
(t-stat)				(7.1201)			(9.6532)	(10.0469)	(8.9888)	(8.8510)
ROE									1.3587	1.5674
(t-stat)									(1.6076)	(2.0191)
Adj-R^2	0.482	0.350	0.549	0.301	0.480	0.565	0.600	0.600	0.521	0.526
RMSE	1.72	1.92	1,60	2	1.72	1.57	1.53	1.53	1.65	1.64
F-test	38	22.4	49.4	18.1	30.4	35.4	33.5	39	44.2	36.4
P-Value	3.79e-22	1.25e-14	9.49e-27	3.33e-12	2.1e-21	1.22e-26	7.18e-28	1.59e-28	1.05e-24	1.81e-24
DOF	155	155	155	155	154	153	152	153	155	154

Table 13: **Value-Weighted Portfolios: Fama-French & Carhartt Factor Models**

Table 13 evaluates the performances of many widely accepted asset pricing models under both the 25 value-weighted SIZE/BM portfolios and 160 value-weighted portfolios, respectively. I test (i) Fama & French (1993, 2015, 2018) three-factor, five-factor, and six-factor model; (ii) the Carhart (1997) four-factor model in the control group. I also report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data are from 1967 to 2019.

Portfolios	Panel A: 25 Value-Size Portfolios				Panel B: 160 Portfolios			
Models	FF3	Carhart 4	FF5	FF6	FF3	Carhart 4	FF5	FF6
Cons.	12.5133	12.3652	12.7910	17.1812	10.2846	11.9407	7.0374	8.9633
(t-stat)	(4.6690)	(5.2685)	(2.6048)	(2.9109)	(7.7313)	(9.3265)	(4.9271)	(5.4820)
Mkt	-5.2035	-5.2421	-5.3605	-9.7129	-2.7481	-4.4886	0.3371	-1.6009
(t-stat)	(-1.8315)	(-2.0693)	(-1.1368)	(-1.6637)	(-1.9764)	(-3.3165)	(0.2332)	(-0.9643)
SMB	2.7571	3.0569	2.5822	2.7828	2.4606	2.5763	2.6080	2.6543
(t-stat)	(4.5520)	(6.9523)	(3.5767)	(5.5847)	(7.7825)	(8.3973)	(8.4905)	(9.2224)
HML	4.1473	3.9557	4.2183	4.1521	3.9052	3.6665	3.2273	3.2036
(t-stat)	(5.8381)	(6.5742)	(6.0234)	(7.0372)	(7.3366)	(7.5167)	(6.5535)	(6.7189)
CMA			3.4854	3.4605			2.0874	1.9810
(t-stat)			(2.4115)	(2.9408)			(3.8656)	(3.7372)
RWM			-0.6106	-2.0594			1.8449	1.8185
(t-stat)			(-0.2676)	(-0.9037)			(4.4326)	(4.5229)
MOM		-10.4177		-10.6653		-5.2230		-4.7712
(t-stat)		(-2.6443)		(-2.9923)		(-3.7991)		(-3.9402)
Adj-R^2	0.680	0.731	0.654	0.719	0.529	0.563	0.584	0.599
RMSE	1.39	1.64	1.45	1.31	1.64	1.58	1.54	1.51
F-test	18	17.3	10.1	11.2	60.5	52.2	45.7	40.5
P-Value	5.15e-06	2.77e-06	7.86e-05	2.99e-05	5.3e-26	8.69e-28	9.54e-29	2.9e-29
DOF	21	20	19	18	156	155	154	153

Table 14: **Volatility Risks in ICAPM: 25 Value-Weighted Portfolios**

Table 14 reports the results of cross-sectional regression of 25 value-weighted SIZE/BM portfolios on various investment-based factor pricing models only driven by productivity uncertainty risks. I then report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data are from 1967 to 2019.

Models	I	II	III	IV	V	VI	VII	VIII
Cons.	6.5614	5.3050	8.7344	3.6796	6.5947	9.1231	10.1450	11.1560
(t-stat)	(3.7850)	(3.3035)	(3.8317)	(2.9190)	(4.3242)	(4.5782)	(4.9048)	(4.5085)
$\Delta\sigma_t$	-0.0010	-0.0017	-0.0016	-0.0015	6.2146e-05	-4.1652e-05	-2.0287e-05	-0.0002
(t-stat)	(-2.9993)	(-6.2129)	(-5.6901)	(-2.8011)	(0.3186)	(-0.1810)	(-0.0829)	(-0.4936)
Q	3.0266				3.6278	3.1156	2.9844	2.8027
(t-stat)	(4.5813)				(5.0535)	(5.1570)	(4.5987)	(3.8833)
EG		2.4684			-3.0640	-3.6743	-2.6023	
(t-stat)		(1.8729)			(-3.3882)	(-3.5091)	(-1.7957)	
Mkt			-0.8767			-1.5322	-2.6216	-3.6880
(t-stat)			(-0.4414)			(-0.7070)	(-1.1687)	(-1.3827)
SMB				2.8774			2.7259	2.8704
(t-stat)				(4.0246)			(4.5978)	(4.3258)
Adj-R^2	0.619	0.465	0.551	0.365	0.697	0.696	0.684	0.676
RMSE	1.52	1.80	1.6	1.96	1.36	1.36	1.39	1.40
F-test	20.5	11.4	15.7	7.89	19.4	14.8	11.4	13.5
P-value	9.35e-06	0.000393	5.78e-05	0.00261	2.95e-06	9.12e-06	3.49e-05	1.71e-05
DOF	22	22	22	22	21	20	19	20

Table 15: **Volatility Risks in ICAPM: 160 Value-Weighted Portfolios**

Table 15 displays the results of second-pass regression of 160 value-weighted stock portfolios on various investment-based factor pricing models only driven by productivity uncertainty risks. I then report the adjusted R^2 and Newey & West (1987) T-statistics with four lags for each estimation. Data are from 1967 to 2019.

Models	I	II	III	IV	V	VI	VII	VIII
Cons.	7.8450	7.3731	10.5010	6.5361	7.5470	9.6418	11.4860	11.9920
(t-stat)	(20.2988)	(15.8600)	(15.4465)	(12.7377)	(16.1213)	(8.3337)	(9.7013)	(10.0037)
$\Delta\sigma_t$	-0.0005	-0.0010	-0.0009	-0.0004	-0.0003	-0.0004	-0.0004	-0.0003
(t-stat)	(-4.4814)	(-6.8308)	(-7.0148)	(-1.9600)	(-2.1428)	(-2.4842)	(-2.5789)	(-1.6503)
Q	2.5108				2.5683	2.1652	2.0422	1.8755
(t-stat)	(7.5291)				(6.8853)	(5.3226)	(5.6501)	(4.4005)
EG		2.1239			-0.3337	-0.8791	3.4461	
(t-stat)		(4.1847)			(-0.5097)	(-1.2028)	(4.0702)	
Mkt			-2.3778			-1.6478	-4.0635	-4.4373
(t-stat)			(-3.3112)			(-1.3810)	(-3.2675)	(-3.5216)
SMB				2.4874			3.0227	2.8051
(t-stat)				(7.2844)			(10.3083)	(9.5027)
Adj-R^2	0.375	0.255	0.328	0.263	0.382	0.392	0.532	0.510
RMSE	1.89	2.06	1.96	2.05	1.88	1.85	1.63	1.67
F-test	48.8	28.2	39.9	29.3	33.8	91.9	37.2	42.4
P-Value	3.4e-17	3.36e-11	9.98e-15	1.48e-11	6.93e-17	7.99e-17	6.92e-25	5.61e-24
DOF	157	157	157	157	156	155	154	155

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